Abstract: When utilizing Performance Based Navigation (PBN) and Automatic Dependent Surveillance Broadcast (ADS-B) it is of particular interest to assess the quality of these new applications. One possible means to analyze the performance is a comparison to position data obtained from a multilateration (MLAT) approach, where various receivers operating at 1090 MHz are employed. This method has the advantage that all airplanes operating under Instrument Flight Rules (IFR) can be tracked through both their Secondary Surveillance Radar (SSR) transponder replies and their transmitted ADS-B messages. For this purpose, a mobile, inexpensive and easy-to-use MLAT system was developed and used for reference tracking. Various field tests revealed that it features a horizontal accuracy of a few meters, outperforming the performance parameter estimation from PBN and ADS-B by an order of magnitude in most cases.

Keywords: Mobile MLAT, PBN, ADS-B, System Performance Assessment

1 INTRODUCTION

In view of worldwide Performance Based Navigation (PBN) and Automatic Dependent Surveillance Broadcast (ADS-B) implementation plans and activities it is of particular interest to assess the quality of these new applications. For instance, within PBN, the Total System Error (TSE) is a safety relevant parameter, especially for applications in demanding environments such as approach procedures within mountainous areas. One solution, in order to estimate the TSE, would be to equip an airplane with a dedicated reference GPS receiver and derive carrier-phase position solutions. This approach would be feasible for flight trials but it is hardly possible to apply it for commercial operations, e.g. for monitoring purposes of a complete traffic situation. The same challenges arise when assessing the quality of ADS-B, where the TSE cannot be derived by the data broadcast, but has to be retrieved on-board through dedicated non-operational equipment.

An alternative approach to assess the TSE or the quality of ADS-B is the use of multilateration (MLAT) techniques to accurately determine the aircraft position at any given time. To this end, skyguide, the Swiss Air Navigation Service Provider (ANSP), asked the Center for Signal Processing and Communication Engineering at the Zurich University of Applied Sciences in Winterthur, Switzerland, to develop a mobile MLAT system. This system, described in more detail in the following section, is based on various portable receiver units (remote units RU) operating at 1090 MHz. This approach is advantageous because all aircraft operating under Instrument Flight Rules (IFR) can be tracked through their transponder replies on Mode-A/C/S. In addition, other messages such as ADS-B telegrams, replies to altitude and ID requests, or arbitrary transmissions from the Airborne Collision Avoidance System (ACAS) can also be utilized. In general, the accuracy of such an MLAT system strongly depends on the accuracy of the time reference at each RU and on the geometric position of each of the RUs. By carrying out both a priori simulations and practical field tests it could be shown that the newly developed MLAT system features a horizontal accuracy in the range of a few meters. In most cases this accuracy is an order of magnitude higher than the horizontal alert limit for most PBN procedures and the navigation accuracy/integrity category for position (NAC\textsubscript{P} and NIC\textsubscript{P}) within ADS-B.

In order to exactly determine the MLAT system’s accuracy dedicated flight trials with aircraft equipped with a Mode-S transponder and an ADS-B transmitter were performed around Zurich Airport. In addition, the aircraft were equipped with independent GNSS receivers which allowed, based on a carrier-phase solution, to determine the position of the aircraft with an accuracy of a few decimeters. These positions were
used as reference in order to assess the mobile MLAT position accuracy. Finally, the attitude information of the aircraft was available through an inertial platform. This is useful in order to analyze any unexpected anomaly or loss of messages which might be caused by aircraft maneuvers. These field tests confirmed the accuracy of the portable MLAT system.

2 MOBILE MLAT HARDWARE

The mobile MLAT receiver, designed by the Centre for Signal Processing and Communications Engineering of the Zurich University of Applied Sciences, serves two main purposes. First, it receives, detects and records the ADS-B and Mode S messages that the various aircraft transmit. Second, it augments the recorded messages by adding both an accurate time stamp indicating at what exact time the message was received and GPS position data indicating the receiver location. The time and position data form the fundament of the Time Difference of Arrival (TDOA) position computation that is explained in more detail in the next section. A block diagram of the receiver hardware is shown in Fig. 1. The receiver’s main components consist of a vertical rod antenna, an analog radio frequency (RF) frontend, an analog-to-digital converter (ADC), a field programmable gate array (FPGA), a GPS module, a microcontroller and a SD card.

Fig. 1: Mobile MLAT receiver hardware block diagram.

In a first step, the aircraft messages are received by an analog RF frontend operating at 1090 MHz. After an analog-to-digital conversion the digital message is passed to an FPGA. Exploiting the fact that each message has a known preamble the FPGA can detect not only the message bits but also the exact time of arrival and adds this time as a time stamp to each message. The time reference for these time stamps stems from a GPS module that is integrated into the receiver hardware. As the MLAT system’s performance is strongly determined by the time stamp accuracy, various efforts were made to obtain a 1-sigma time stamp accuracy as high as 21 ns, which in turn corresponds to a distance difference of 6.3 m. In a next step, the FPGA passes the digital message to a microcontroller that stores it on the built-in SD card. At present, the card has a storage capacity of 32 GB corresponding to roughly 650 million ADS-B messages (including the time stamps). The rate at which messages are received is strongly determined by the receiver location. For areas with good RF reception and large traffic volume, e.g., close by airports, the message rate can easily exceed 4’000 messages per second. Various field tests around Zurich Airport revealed that such SD cards allow for roughly 6 days of non-stop message recording. To easily access the SD card’s data the receiver hardware is equipped with a USB port that allows plug-and-play connection with any PC or laptop. Furthermore, the receiver hardware also features an Ethernet port such that, instead of storing all received messages on the SD card, they can directly be transmitted over the Internet. This is of particular importance when long-term applications for several weeks or months are planned. In addition to the relatively low price, one of the system’s main advantages is its mobility. The receiver units are designed in such way that a single person can easily carry them. To this end, all electronic components are integrated into a waterproof and shock-resistant housing featuring a total mass of some 6 kg, whereas the receiver antenna with a total length of roughly 50 cm is mounted on a portable aluminum tripod. For mobile use the receiver hardware can be connected to a portable battery pack allowing for at least 48 hours of non-stop operation, even for temperatures below 0 °C. The receiver electronics, together with the waterproof housing, are depicted in Fig. 2. An image of the entire mobile system is shown in the appendix of this paper.

At present, the receivers are primarily used for off-line multilateration. This means that whenever the system is running each receiver stores all incoming messages on the built-in SD card. In a later step all of these messages are copied to a single PC or laptop, and this is where the multilateration is then carried out. Directly transmitting the received data over the
Internet would allow for quasi real-time multilateration. Yet, this feature has not been implemented at the present stage.

3 POSITION ESTIMATION

The Total System Error (TSE) is an important and safety relevant system performance parameter. Its value describes the difference between the desired and the actual position or track of a vehicle. In principle, the TSE consists mainly of the Navigation System Error (NSE) and Flight Technical Error (FTE). Additionally, the position accuracy of the mobile MLAT has to be taken into account when determining the TSE through this system. Fig. 3 depicts the relation of these errors. When assuming unbiased and Gaussian distributed errors, then its relation is described through the sum of the squares of the errors.

\[ \text{Fig. 3: Relation of TSE, FTE, NSE and mobile MLAT position error.} \]

Consequently, in a first step, the position determination algorithms are discussed. In a second step, the a priori and an a posteriori accuracy of the mobile MLAT position determination is assessed.

The employed MLAT system computes the aircraft position with the aid of the Time Difference of Arrival (TDOA). This method, also known as hyperbolic location estimation, is based on the time differences at which a signal transmitted by the aircraft arrives at each of the RUs. The observation equation for unperturbed TDOA within a topographic Cartesian coordinate system reads

\[ d'_{jk} = \Delta t_{jk} \cdot c = d'_{p_j} - d'_{p_k} \]  \hspace{1cm} (1)

with

\[ d'_{p_j} = \sqrt{(x_p - x_j)^2 + (y_p - y_j)^2 + (z_p - z_j)^2} \]  \hspace{1cm} (2)

where

\[ d'_{jk}: \text{difference of geometrical distance between aircraft and } j^{th}, \text{respectively} \ k^{th} \text{ RU} \]

\[ \Delta t_{jk}: \text{TDOA between } j^{th} \text{ and } k^{th} \text{ RU} \]

\[ c: \text{speed of light} \]

\[ d'_{p_j}: \text{geometrical distance between aircraft and } j^{th} \text{ RU} \]

\[ d'_{pk}: \text{geometrical distance between aircraft and } p^{th} \text{ RU} \]

\[ x_p, y_p, z_p: \text{unknown position of aircraft} \]

\[ x_j, y_j, z_j: \text{known position of } j^{th} \text{ RU} \]

\[ x_k, y_k, z_k: \text{known position of } k^{th} \text{ RU}. \]

Equation (1) holds only for error free measurements. Main errors affecting MLAT are RU clock synchronization, signal propagation and white noise [1]. Synchronization and clock errors common to all RU are eliminated due to receiving time differencing. All other errors are assumed to be gaussian distributed with zero mean error and denoted \( \epsilon_i \). It follows for the observed difference of distances

\[ d_{jk} = d'_{jk} + \epsilon_{jk} \]  \hspace{1cm} (4)

with

\[ \epsilon_{jk}: \text{difference of distance measurement errors between the } j^{th} \text{ and } k^{th} \text{ RU}. \]

An non-linear equation system with the distance differences \( d_{jk} \) based on TDOA observations, the known RU positions and the unknown aircraft position is therefore defined. Linearization of observations, e.g. by means of Taylor series, leads to the following linear equation for the position vector \( \vec{x} \)

\[ A\vec{x} = \vec{d} \]  \hspace{1cm} (5)

with

\[ \vec{x} = (x_p, y_p)^T \text{ or } (x_p, y_p, z_p)^T: \text{unknown aircraft position vector} \]

\[ A: \text{design matrix} \]

\[ \vec{d} = (d_{jk}, \ldots)^T: \text{known observations vector}. \]
The vector $\tilde{x}$ can be two dimensional ($D = 2$) for purely horizontal position solutions or three dimensional ($D = 3$) when additionally estimating the altitude. For quality purposes the number of observations $n$ is generally larger than the number of unknowns $D$. The relation between the number of RU $m$ and the number of observations $n$ is given by

$$n = \frac{m!}{2!(m-2)!}, \quad m \geq 2$$

and therefore

$$n > D.$$  \hfill (7)

In this case, the overdetermined equation (5) has to be expanded and solved for $\tilde{x}$

$$\tilde{x} = (A^T A)^{-1} A^T \tilde{d}$$ \hfill (8)

The solution (8) holds for minimum condition of the observation residuals $\tilde{v} = (v_{jk}, \ldots)$ and is known as least square adjustment based on the Gauss-Newton method. Equation (4) is rewritten as

$$d_{jk} = \tilde{d}_{jk} + v_{jk}$$ \hfill (9)

with

$$\tilde{d}_{jk}: \text{adjusted observations.}$$

A weight matrix $P$ can be used in order to characterize different error contributions of the signal propagation and RU:

$$\tilde{x} = (A^T PA)^{-1} A^T P \tilde{d}.$$ \hfill (10)

### 4 A PRIORI POSITION ERROR

The design matrix $A$ includes the geometry information of the RU related to the aircraft position and is independent of the observations. Therefore an a priori estimation of the position errors is possible. The cofactor matrix $Q_s$ of the unknown aircraft position $\tilde{x}$ is

$$Q_s = (A^T PA)^{-1}. $$ \hfill (11)

Different Dilution of Precision (DOP) values are derived through the diagonal elements of $Q_s$. The following equations hold for $D = 3$.

$$\begin{align*}
\text{HDOP} &= \sqrt{q_{xx} + q_{yy}} \\
\text{VDOP} &= \sqrt{q_{zz}} \\
\text{PDOP} &= \sqrt{q_{xx} + q_{yy} + q_{zz}}
\end{align*}$$ \hfill (12)

where $q_{xx}$, $q_{yy}$ and $q_{zz}$ are the diagonal elements of $Q_s$.

An a priori position error estimation is achieved by multiplication of the DOP values with the expected observation errors given as a distance. These are characterized through the assumed zero mean gaussian distribution and are composed by clock synchronization, signal propagation and white noise. Assuming that clock synchronization and white noise errors are identical for each observed RU pair at each epoch, these may be treated separately from the signal propagation error. The latter could for instance be considered within the weight matrix $P$. Therefore

$$\sigma_r = \sqrt{\sigma_3^2 + \sigma_n^2}$$ \hfill (13)

with

$$\begin{align*}
\sigma_r: \text{standard deviation of clock synchronisation error} \\
\sigma_n: \text{standard deviation of white noise} \\
\sigma_c: \text{resulting standard deviation.}
\end{align*}$$

For the three-dimensional case the a priori horizontal position error reads

$$\sigma_{xy} = \sigma_r \cdot c \cdot \text{HDOP}. $$ \hfill (14)

Fig. 4 shows a MLAT layout based on 4 RU (triangles) which was used for first trials around Zurich Airport. The dotted line depicts the approach path to runway 14 which was of main interest during these trials. The represented area has a size of 30 km by 30 km. It covers also the region to the north east due to the planned trial flight pattern. The a priori horizontal position errors are calculated for a constant altitude at 4000 ft above mean sea level (AMSL). According to the tests within the laboratory, the clock accuracy including white noise is determined to be $\sigma_c = 21$ ns. Therefore

$$\sigma_r = \sqrt{2}\sigma_c$$ \hfill (15)

for any pair of RU. Applying equation (14) yields the a priori horizontal position error depicted in Fig. 4 as contour lines. The unit is meters.

In addition to all previous considerations, the signal propagation is also obstructed due to terrain, vegetation, buildings and other obstacles. At a frequency of 1090 MHz, the signal obstruction can be approximated by line of sight methods. If these effects are taken into consideration, based on a digital surface model with a resolution of some 20 m, they result in a partially decreased position performance as shown in Fig. 5.
In cases where only along track or cross track position errors are of interest it is advantageous to analyze more in depth the a priori errors related to the azimuth of the desired flight track. This can be achieved by assessing two dimensional Gaussian distributions and deriving error ellipses, which describes the horizontal position error characteristics analogously to the standard deviation for one dimensional Gaussian distributions. Fig. 6 depicts an aircraft with the desired track defined through the azimuth $\alpha$. The error ellipse at the position of the aircraft, which depends on the geometry of the RU and the observation accuracies, is described by the semi-major axis $a$, semi-minor axis $b$ and the orientation $\omega$. The a priori cross-track error $\sigma_{ct}$ corresponds to the distance between the tangent to the ellipse parallel to the desired flight track. The error probability distribution in cross-track direction is again Gaussian distributed. The along-track error $\sigma_{at}$ is derived analogously with the tangent perpendicular to the desired flight track. The probability, that the estimated position lies within the ellipse is 0.394 (or 0.865 when calculating with $2\sigma$ values).

Optimum RU layout is achieved, when $a$ or $b$ is aligned with the desired flight track, respectively when the difference between $\alpha$ and $\omega$ is 0 or $\pi/2$. This optimization is useful when assessing new flight procedures where the desired flight track is known. The parameters of the error ellipse are derived by following equations

\[
\begin{align*}
a &= \sigma_r \sqrt{\frac{q_{xx} + q_{yy}}{2} + \sqrt{\left(\frac{q_{xx} - q_{yy}}{2}\right)^2 + q_{xy}^2}} \\
b &= \sigma_r \sqrt{\frac{q_{xx} + q_{yy}}{2} - \sqrt{\left(\frac{q_{xx} - q_{yy}}{2}\right)^2 + q_{xy}^2}} \\
\omega &= \frac{1}{2} \cdot \tan^{-1} \left(\frac{2q_{xy}}{q_{xx} - q_{yy}}\right)
\end{align*}
\]

where $q_{xx}$ and $q_{yy}$ are the diagonal elements of $Q_x$ and $q_{xy}$ is the corresponding non-diagonal element.

5 FIELD TESTS AND A POSTERIORI POSITION ERROR

In order to assess the mobile system’s accuracy we compare the horizontal aircraft position obtained from the MLAT system with high-accuracy GPS position data. These data stem from a calibration flight with a Hawker Beech King Air 350, that was conducted by Flight Calibration Services (FCS) in order to calibrate an Instrument Landing System (ILS) at Zurich Airport. The aircraft was equipped with independent GNSS receivers that allowed, based on a
carrier-phase solution, to determine the aircraft location with an accuracy of a few decimeters. This GPS reference track was then compared to the aircraft position obtained from the mobile MLAT system, and the results are depicted in Fig. 7. Due to the large amount of data available we restrict ourselves to one representative ILS approach that covers a time span of some 140 s.

The absolute horizontal position errors, i.e., the difference between the MLAT solution and the GPS data, for this flight are represented as black dots on the top part of Fig. 7. The black line indicates the expected (a priori) horizontal position error derived according to Equation (14). The vertical profile is shown on the bottom of Fig. 7. Within the first part of the approach the a priori position error estimation is slightly too optimistic. This might have different reasons. First, the statistical model used for the least squares adjustment assumes, that all observation errors are independent. This assumption does not necessarily hold for TDOA based systems and therefore might have an impact on the quality of the position error estimations. However, these estimations adequately describe the position errors within the perimeter defined by the RU. Within this area a mean horizontal position error of 5.4 m, respectively a 95 % -quantile $q_{0.95} = 11$ m is measured. The statistical distribution of the horizontal position error is shown in Fig. 8. Second, signal attenuation and multipath effects might impact the quality of the received signal. Yet, these effects have not been investigated in detail but will be analyzed in depth in future work.

6 APPLICATION EXAMPLES

In the following, we will briefly discuss two potential applications of the mobile MLAT system.

6.1 Approach Procedure

Various recordings from traffic of opportunity, combined with MLAT measurements, were used to determine the cross track error on an approach procedure. In the first example, eight approaches to runway 14 of Zurich Airport were recorded. The aircraft are mostly medium-range commercial passenger types. Although GBAS CAT-I and RNAV approach procedures are operational on this runway, the recorded flights were performed with use of an ILS. Nevertheless, these approaches can be used to demonstrate the application of the mobile MLAT.

Fig. 9: Cross track error of eight flights of opportunity on approach to runway 14 at Zurich Airport.
Fig. 9 shows the cross track error of all eight flights depending on the distance to the runway threshold (THR). The flight direction within the figure is from right to left. The error is composed by the TSE and the mobile MLAT position error. The latter has to be subtracted in order to estimate the correct TSE. As a rule of thumb, the accuracy of the reference track should be at least an order of magnitude better than the TSE. This is hardly the case within this example. However, in the frame of PBN it is of interest to compare the TSE with the alert limit corresponding to the procedure being assessed, e.g. 0.3NM (556m) for general RNAV approaches [2]. This can easily be achieved with the mobile MLAT system.

6.2 Assessment of ADS-B Performance

The second example focuses on the assessment of ADS-B performance. ADS-B regularly broadcast the vehicle position. In contrast to classical surveillance systems such as Secondary Surveillance Radar (SSR), an ANSP has to rely on the position performance provided by the vehicle. The performance of the vehicle’s position data is obtained from various parameters such as the Navigation Accuracy Category for Position (NAC_P) and Navigation Integrity Category for Position (NIC_P) [3][4]. Thus, it is crucial to assess the quality of these performance parameters. The mobile MLAT system allows to derive an independent vehicle position by multilateration of the ADS-B squitter signals. Additionally, the vehicle’s position provided through the ADS-B is decoded and compared with position calculated by the mobile MLAT system. Finally, this difference is compared with the NAC_P and NIC_P and the quality of these performance parameters can be evaluated. Fig. 10 depicts two different tracks of the same flight during an approach procedure. The black line to the left is derived from the position message encoded within the ADS-B squitters. The gray line to the right shows the track derived through multilateration. The desired track is shown as black dotted line. The offset is clearly visible. The black and gray dot on the top of the figure shows the position at the same epoch. Therefore the horizontal position provided through the ADS-B has an error of 450m. The corresponding performance parameters provided by the ADS-B indicate a horizontal accuracy larger than 10 NM and an unknown integrity level. Therefore these parameters conform in this case to the horizontal position accuracy provided by the ADS-B-.

7 CONCLUSIONS

A mobile, inexpensive, autonomous and easy-to-use MLAT system is presented. The objective of this system is to allow accurate position estimation of all IFR traffic within an area of interest in order to assess the performance of different systems such as PBN and ADS-B. A priori and a posteriori horizontal position accuracy of the mobile MLAT system are discussed. Based on flight trials performed with a Beech King Air 350 of Flight Calibration Services it is shown, that the MLAT system achieves position accuracies as high as 5m inside the central area covered by the RUs. Finally, possible applications for the mobile MLAT system in the frame of PBN and ADS-B are presented.

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APPENDIX

A complete receiver unit of the mobile MLAT system is shown in Fig. 11. It comprises a rod antenna mounted on a tripod, the receiver electronics inside a waterproof housing and a battery pack allowing for at least 48 h of autonomous operation. The entire system can easily be carried and assembled by one single person.
References


