Yes, we CANN!

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SAV Actuarial Data Science Working Party

- **Actuarial Data Science** Initiative of the Swiss Association of Actuaries SAV
  - Case study: French motor third-party liability claims (2018)
  - Unsupervised learning: What is a sports car? (2019)

▷ these tutorials are available from www.ssrn.com
▷ all code (and data) is available from GitHub

- For more information, see:
  
  www.actuarialdatascience.org
Yes, we CANN!
The modeling cycle

(1) data collection, data cleaning and data pre-processing (80% of total time)

(2) selection of model class (data or algorithmic model, Breiman 2001)

(3) choice of objective function

(4) ’solving’ a (non-convex) optimization problem

(5) model validation

(6) possibly go back to (1)

▷ ’solving’ involves:

★ choice of algorithm
★ choice of stopping criterion, step size, etc.
★ choice of seed (starting value)
Car insurance frequency example

> str(freMTPL2freq)  #source R package CASdatasets
'data.frame': 678013 obs. of 12 variables:
  $ IDpol : num 1 3 5 10 11 13 15 17 18 21 ...  
  $ ClaimNb : num 1 1 1 1 1 1 1 1 1 1 ...  
  $ Exposure : num 0.1 0.77 0.75 0.09 0.84 0.52 0.45 0.27 0.71 0.15 ...  
  $ Area  : Factor w/ 6 levels "A","B","C","D",...: 4 4 2 2 2 5 5 3 3 2 ...  
  $ VehPower : int 5 5 6 7 7 6 6 7 7 7 ...  
  $ VehAge  : int 0 0 2 0 2 0 2 0 0 0 ...  
  $ DrivAge : int 55 55 52 46 46 38 38 33 33 41 ...  
  $ BonusMalus: int 50 50 50 50 50 50 50 68 68 50 ...  
  $ VehBrand : Factor w/ 11 levels "B1","B10","B11",...: 4 4 4 4 4 4 4 4 4 4 4 ...  
  $ VehGas  : Factor w/ 2 levels "Diesel","Regular": 2 2 1 1 1 2 2 1 1 1 ...  
  $ Density : int 1217 1217 54 76 76 3003 3003 137 137 60 ...  
  $ Region : Factor w/ 22 levels "R11","R21","R22",...: 18 18 3 15 15 8 8 20 20 12 ...  

observed frequencies per regional groups
observed frequency per driver's age groups
observed frequency per car brand groups
Generalized linear models (GLMs)

- Determine from data \( \mathcal{D} = \{(Y_1, x_1), \ldots, (Y_n, x_n)\} \) an unknown regression function

\[
\mu(x) = \mathbb{E}[Y].
\]

- Selection of model class: Poisson GLM with canonical (log-)link:

\[
x \mapsto \mu_{\beta}^{\text{GLM}}(x) = \exp\langle \beta, x \rangle = \exp \left\{ \beta_0 + \sum_j \beta_j x_j \right\}.
\]

- Estimate regression parameter \( \beta \) with maximum likelihood \( \hat{\beta}^{\text{MLE}} \) by minimizing the corresponding deviance loss (objective function)

\[
\beta \mapsto \mathcal{L}_{\mathcal{D}}(\beta).
\]
Example: car insurance Poisson frequencies

After pre-processing the covariates $x$:

<table>
<thead>
<tr>
<th>Model</th>
<th>$# \text{ param.}$</th>
<th>\text{in-sample loss (in } 10^{-2})</th>
<th>\text{out-of-sample loss (in } 10^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>homogeneous ($\mu \equiv \text{const.}$)</td>
<td>1</td>
<td>32.935</td>
<td>33.861</td>
</tr>
<tr>
<td>Model GLM (Poisson)</td>
<td>48</td>
<td>31.257</td>
<td>32.149</td>
</tr>
</tbody>
</table>

Note for low frequency examples of, say, 5%: we have in the true model $\mathcal{L}_D \approx 30.3 \cdot 10^{-2}$.

- This convex optimization problem has a unique optimal solution.
- The solution satisfies the balance property (under the canonical link choice)

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \exp\langle \hat{\beta}_{\text{MLE}}, x_i \rangle.$$
From GLMs to neural networks

- Example of a GLM (with log-link $\Rightarrow$ exponential output activation):

$$x \mapsto \mu^\text{GLM}_\beta(x) = \exp \langle \beta, x \rangle.$$ 

- Choose network of depth $d \in \mathbb{N}$ with network parameter $\theta = (\theta_{1:d}, \theta_{d+1})$:

$$x \mapsto \mu^\text{NN}_\theta(x) = \exp \langle \theta_{d+1}, z \rangle,$$

with neural network function (covariate pre-processing $x \mapsto z$)

$$x \mapsto z = z^{\text{(d:1)}}_{\theta_{1:d}}(x) = \left( z^{(d)} \circ \cdots \circ z^{(1)} \right)(x).$$
Neural network with embeddings

- Network of depth \( d \in \mathbb{N} \) with network parameter \( \theta \)

\[
\mathbf{x} \mapsto \mu_\theta^{\text{NN}}(\mathbf{x}) = \exp \langle \theta_{d+1}, \mathbf{z} \rangle = \exp \langle \theta_{d+1}, \left( z^{(d)} \circ \cdots \circ z^{(1)} \right)(\mathbf{x}) \rangle.
\]

- Gradient descent method (GDM) provides \( \hat{\theta} \) w.r.t. deviance loss \( \theta \mapsto \mathcal{L}_D(\theta) \).

- Exercise early stopping of GDM because MLE over-fits (in-sample).
Remarks on the neural network approach

+ Use embedding layers for categorical variables.

+ Typically, the neural network outperforms the GLM approach in terms of out-of-sample prediction accuracy.

− Resulting prices are not unique, but depend on seeds.

− The neural network does not build on improving the GLM.

− The neural network fails to have the balance property.
• Choose regression function with parameter \((\beta, \theta)\)

\[
\mathbf{x} \mapsto \mu_{(\beta, \theta)}^{\text{CANN}}(\mathbf{x}) = \exp \left\{ \langle \beta, \mathbf{x} \rangle + \langle \theta_{d+1}, \left( z^{(d)} \circ \cdots \circ z^{(1)} \right)(\mathbf{x}) \rangle \right\}.
\]

• GDM provides \((\hat{\beta}, \hat{\theta})\) w.r.t. deviance loss \((\beta, \theta) \mapsto \mathcal{L}_D(\beta, \theta)\).
Combined Actuarial Neural Network: part II

- Choose regression function with parameter $(\beta, \theta)$

\[
\mu_{(\beta, \theta)}^{CANN}(\bm{x}) = \exp \left\{ \langle \beta, \bm{x} \rangle + \langle \theta_{d+1}, (z^{(d)} \circ \cdots \circ z^{(1)})(\bm{x}) \rangle \right\}.
\]

- GDM provides $(\hat{\beta}, \hat{\theta})$ w.r.t. deviance loss $(\beta, \theta) \mapsto \mathcal{L}_D(\beta, \theta)$.

- Initialize gradient descent algorithm with $\hat{\beta}^{MLE}$ and $\theta_{d+1} = 0!$
Possible GDM results of the CANN approach.
CANN example: car insurance frequencies

<table>
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</tr>
<tr>
<td>CANN (2-dim. embeddings)</td>
<td>792 (+48)</td>
<td>30.476</td>
<td>31.566</td>
</tr>
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Note for low frequency examples of, say, 5%: we have in the true model $\mathcal{L}_D \approx 30.3 \cdot 10^{-2}$. 
Variants of CANN

- Freeze $\hat{\beta}^{\text{MLE}}$ (use as offset) and only train network parameter $\theta = (\theta_{1:d}, \theta_{d+1})$

$$\mu_{(\beta, \theta)}^{\text{CANN}}(x) = \exp \left\{ \langle \hat{\beta}^{\text{MLE}}, x \rangle + \langle \theta_{d+1}, (z^{(d)} \circ \cdots \circ z^{(1)}) (x) \rangle \right\}.$$ 

- Introduce trainable credibility weight $\alpha$ for the offset

$$\mu_{(\beta, \theta)}^{\text{CANN}}(x) = \exp \left\{ \alpha \langle \hat{\beta}^{\text{MLE}}, x \rangle + (1 - \alpha) \langle \theta_{d+1}, (z^{(d)} \circ \cdots \circ z^{(1)}) (x) \rangle \right\}.$$ 

- Find missing interactions in $(x_l, x_k)$ in addition to the offset

$$\mu_{(\beta, \theta)}^{\text{CANN}}(x) = \exp \left\{ \langle \hat{\beta}^{\text{MLE}}, x \rangle + \langle \theta_{d+1}, (z^{(d)} \circ \cdots \circ z^{(1)}) (x_l, x_k) \rangle \right\}.$$
Regularization step for the balance property

• Neural network calibrations do not have the balance property, yet.

• Apply an additional GLM step on the learned representation

\[ x \mapsto z = z(x) = (z^{(d)} \circ \ldots \circ z^{(1)}) \circ (x), \]

keeping the offset \( \langle \hat{\beta}^{\text{MLE}}, x \rangle \) fixed, i.e. calculate MLE \( \hat{\theta}_{d+1}^{\text{MLE}} \) of regression function

\[ z = z(x) \mapsto \exp \left\{ \langle \hat{\beta}^{\text{MLE}}, x \rangle + \langle \theta_{d+1}, z \rangle \right\}. \]

• This regularization step is important, in particular, in classification problems having the class imbalance problem!
Summary and outlook

- CANN allows us to identify missing structure in GLMs (more) explicitly.
- An additional GLM step allows us to satisfy the balance property.
- CANN allows us to learn across different portfolios.

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