Yes, we CANN!

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SAV Actuarial Data Science Working Party

- **Actuarial Data Science** Initiative of the Swiss Association of Actuaries SAV
  - ⋆ Case study: French motor third-party liability claims (2018)
  - ⋆ Insights from inside neural networks (2018)
  - ⋆ Nesting classical actuarial models into neural networks (2019)
  - ⋆ On boosting: theory and applications (2019)
  - ⋆ Unsupervised learning: What is a sports car? (2019)
  - ⋆ Lee and Carter go machine learning: recurrent neural networks (2019)

    - ▷ these tutorials are available from `www.ssrn.com`
    - ▷ all code (and data) is available from GitHub

- For more information, see:

`www.actuarialdatascience.org`

Yes, we CANN!

The modeling cycle

(1) data collection, data cleaning and data pre-processing (80% of total time)

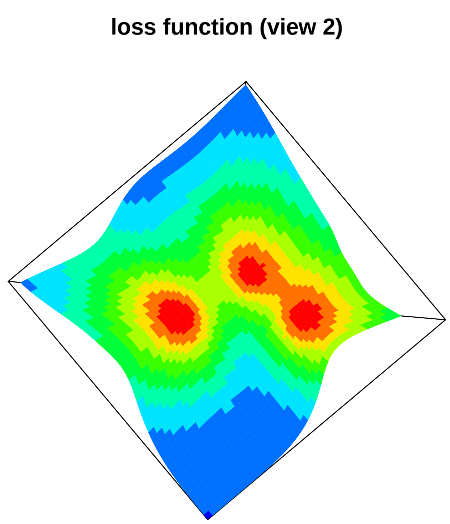
(2) selection of model class (data or algorithmic model, Breiman 2001)

(3) choice of objective function

(4) 'solving' a (non-convex) optimization problem

(5) model validation

(6) possibly go back to (1)

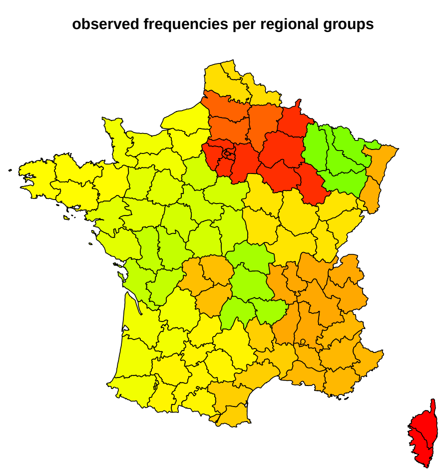


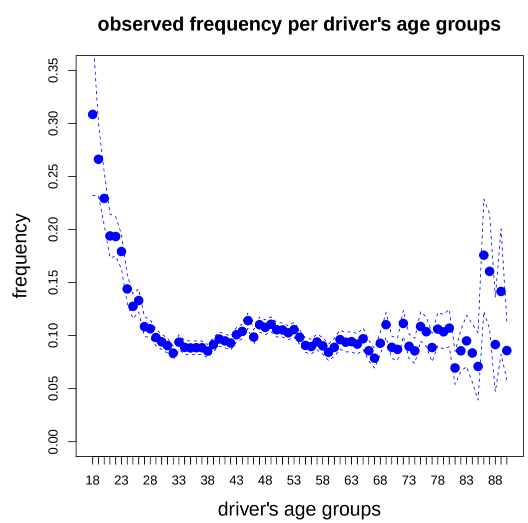
▷ 'solving' involves:

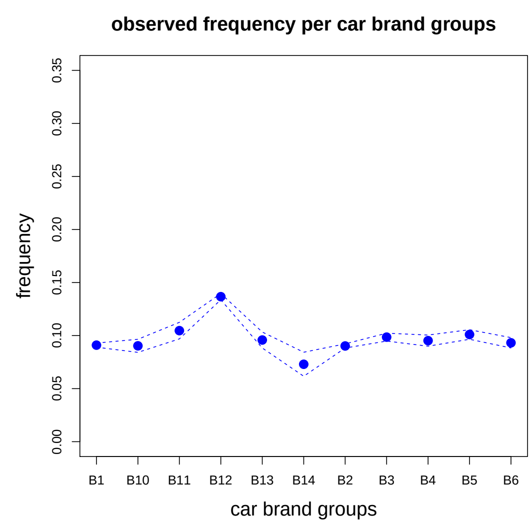
⋆ choice of algorithm
⋆ choice of stopping criterion, step size, etc.
⋆ choice of seed (starting value)

Car insurance frequency example

```
> str(freMTPL2freq)        #source R package CASdatasets
'data.frame':    678013 obs. of  12 variables:
 $ IDpol      : num  1 3 5 10 11 13 15 17 18 21 ...
 $ ClaimNb    : num 1 1 1 1 1 1 1 1 1 1 ...
 $ Exposure   : num  0.1 0.77 0.75 0.09 0.84 0.52 0.45 0.27 0.71 0.15 ...
 $ Area       : Factor w/ 6 levels "A","B","C","D",..: 4 4 2 2 2 5 5 3 3 2 ...
 $ VehPower   : int  5 5 6 7 7 6 6 7 7 7 ...
 $ VehAge     : int  0 0 2 0 0 2 2 0 0 0 ...
 $ DrivAge    : int  55 55 52 46 46 38 38 33 33 41 ...
 $ BonusMalus: int  50 50 50 50 50 50 50 68 68 50 ...
 $ VehBrand   : Factor w/ 11 levels "B1","B10","B11",..: 4 4 4 4 4 4 4 4 4 4 ...
 $ VehGas     : Factor w/ 2 levels "Diesel","Regular": 2 2 1 1 1 2 2 1 1 1 ...
 $ Density    : int  1217 1217 54 76 76 3003 3003 137 137 60 ...
 $ Region     : Factor w/ 22 levels "R11","R21","R22",..: 18 18 3 15 15 8 8 20 20 12 ...
```







Generalized linear models (GLMs)

- Determine from data $\mathcal{D} = \{(Y_1, \boldsymbol{x}_1), \ldots, (Y_n, \boldsymbol{x}_n)\}$ an unknown regression function

$$\boldsymbol{x} \;\mapsto\; \mu(\boldsymbol{x}) = \mathbb{E}[Y].$$

- Selection of model class: Poisson GLM with canonical (log-)link:

$$\boldsymbol{x} \;\mapsto\; \mu_{\boldsymbol{\beta}}^{\mathrm{GLM}}(\boldsymbol{x}) \;=\; \exp\langle \boldsymbol{\beta}, \boldsymbol{x} \rangle \;=\; \exp\left\{ \beta_0 + \sum_j \beta_j x_j \right\}.$$

- Estimate regression parameter $\boldsymbol{\beta}$ with maximum likelihood $\widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}$ by minimizing the corresponding deviance loss (objective function)

$$\boldsymbol{\beta} \;\mapsto\; \mathcal{L}_{\mathcal{D}}(\boldsymbol{\beta}).$$

Example: car insurance Poisson frequencies

After pre-processing the covariates $\boldsymbol{x}$:

	# param.	in-sample loss (in 10^{-2})	out-of-sample loss (in 10^{-2})
homogeneous ($\mu \equiv \text{const.}$)	1	32.935	33.861
Model GLM (Poisson)	48	31.257	32.149

Note for low frequency examples of, say, 5%: we have in the true model $\mathcal{L}_{\mathcal{D}} \approx 30.3 \cdot 10^{-2}$.

- This convex optimization problem has a unique optimal solution.
- The solution satisfies the balance property (under the canonical link choice)

$$\sum_{i=1}^{n} Y_i \;=\; \sum_{i=1}^{n} \exp\langle \widehat{\boldsymbol{\beta}}^{\text{MLE}}, \boldsymbol{x}_i \rangle.$$

From GLMs to neural networks

- Example of a GLM (with log-link $\Rightarrow$ exponential output activation):

$$\boldsymbol{x} \;\mapsto\; \mu_{\boldsymbol{\beta}}^{\mathrm{GLM}}(\boldsymbol{x}) \;=\; \exp\langle\boldsymbol{\beta}, \boldsymbol{x}\rangle.$$

- Choose network of depth $d \in \mathbb{N}$ with network parameter $\theta = (\theta_{1:d}, \theta_{d+1})$:

$$\boldsymbol{x} \;\mapsto\; \mu_{\theta}^{\mathrm{NN}}(\boldsymbol{x}) \;=\; \exp\langle\theta_{d+1}, \boldsymbol{z}\rangle,$$

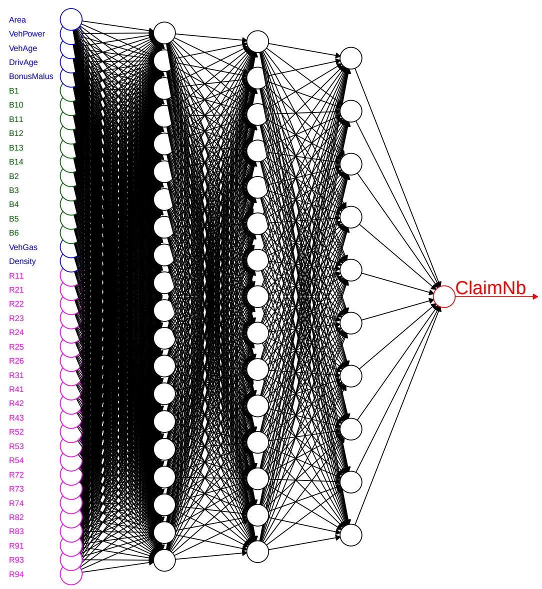
with neural network function (covariate pre-processing $\boldsymbol{x} \mapsto \boldsymbol{z}$)

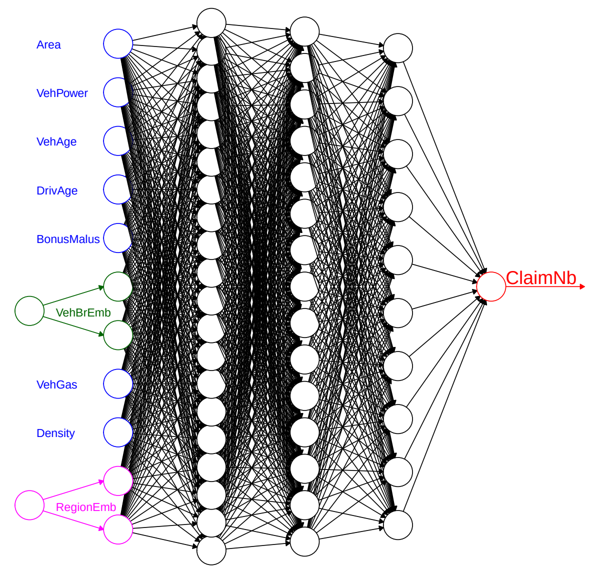
$$\boldsymbol{x} \;\mapsto\; \boldsymbol{z} \;=\; \boldsymbol{z}_{\theta_{1:d}}^{(d:1)}(\boldsymbol{x}) \;=\; \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x}).$$

Neural network with embeddings

- Network of depth $d \in \mathbb{N}$ with network parameter θ

$$\boldsymbol{x} \;\mapsto\; \mu_\theta^{\rm NN}(\boldsymbol{x}) \;=\; \exp\left\langle \theta_{d+1}, \boldsymbol{z}\right\rangle \;=\; \exp\left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x})\right\rangle.$$





- Gradient descent method (GDM) provides $\widehat{\theta}$ w.r.t. deviance loss $\theta \mapsto \mathcal{L}_{\mathcal{D}}(\theta)$.

- Exercise early stopping of GDM because MLE over-fits (in-sample).

Remarks on the neural network approach

+ Use embedding layers for categorical variables.

+ Typically, the neural network outperforms the GLM approach in terms of out-of-sample prediction accuracy.

– Resulting prices are not unique, but depend on seeds.

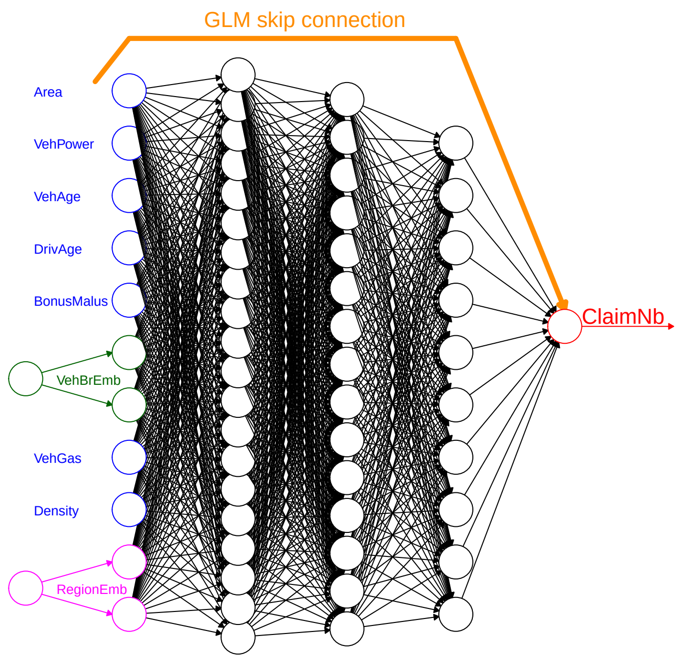
– The neural network does not build on improving the GLM.

– The neural network fails to have the balance property.

Combined Actuarial Neural Network: part I

- Choose regression function with parameter $(\boldsymbol{\beta}, \theta)$

$$\boldsymbol{x} \;\mapsto\; \mu^{\text{CANN}}_{(\boldsymbol{\beta},\theta)}(\boldsymbol{x}) = \exp\left\{\langle \boldsymbol{\beta}, \boldsymbol{x}\rangle + \left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x})\right\rangle\right\}.$$

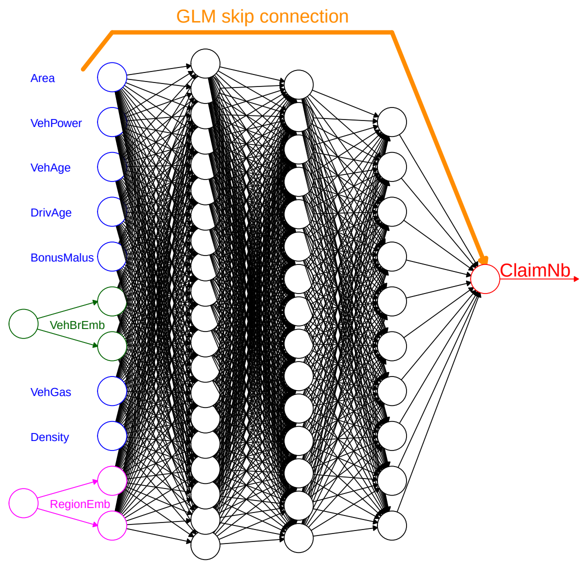


- GDM provides $(\widehat{\boldsymbol{\beta}}, \widehat{\theta})$ w.r.t. deviance loss $(\boldsymbol{\beta}, \theta) \mapsto \mathcal{L}_{\mathcal{D}}(\boldsymbol{\beta}, \theta)$.

Combined Actuarial Neural Network: part II

- Choose regression function with parameter $(\boldsymbol{\beta}, \theta)$

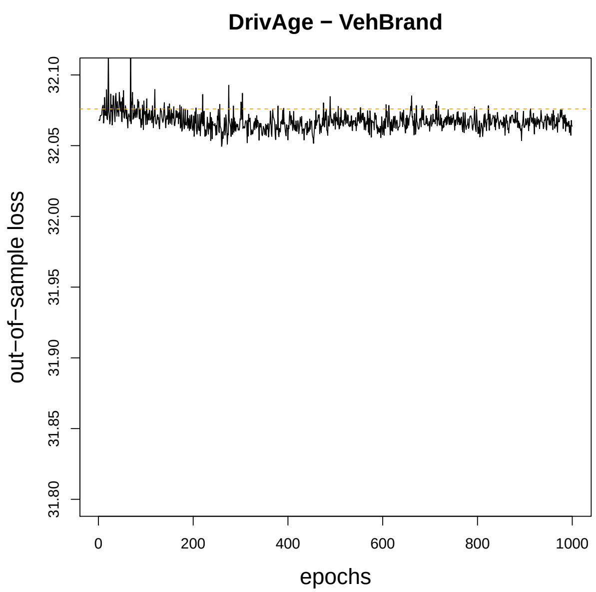
$$\mu^{\text{CANN}}_{(\boldsymbol{\beta},\theta)}(\boldsymbol{x}) = \exp\left\{\langle \boldsymbol{\beta}, \boldsymbol{x}\rangle + \left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x})\right\rangle\right\}.$$

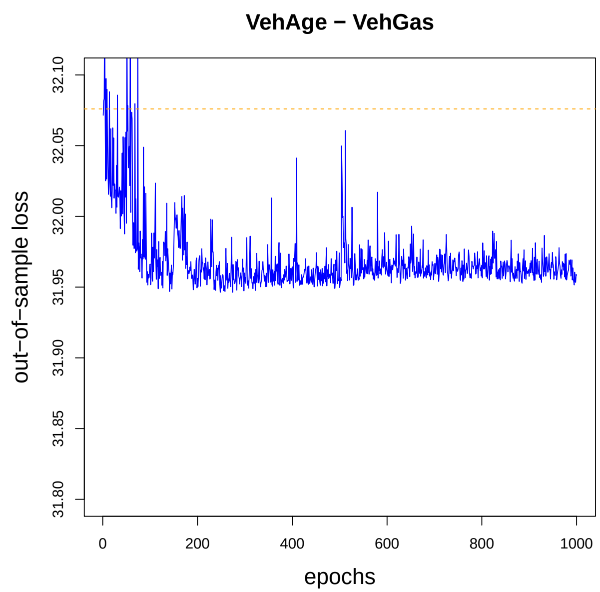


- GDM provides $(\widehat{\boldsymbol{\beta}}, \widehat{\theta})$ w.r.t. deviance loss $(\boldsymbol{\beta}, \theta) \mapsto \mathcal{L}_{\mathcal{D}}(\boldsymbol{\beta}, \theta)$.

- Initialize gradient descent algorithm with $\widehat{\boldsymbol{\beta}}^{\text{MLE}}$ and $\theta_{d+1} = 0$!

Combined Actuarial Neural Network



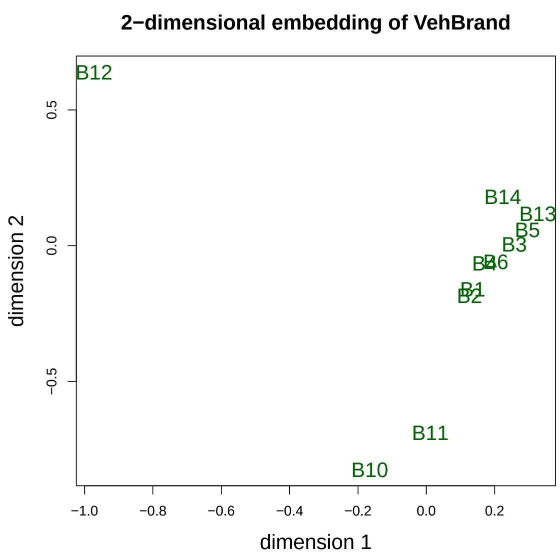


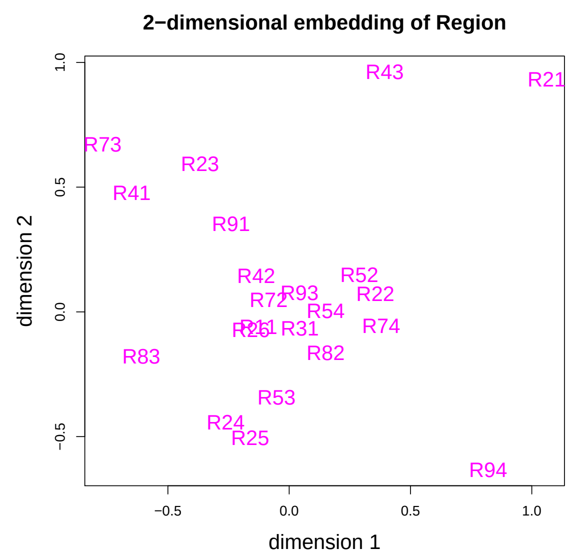
Possible GDM results of the CANN approach.

CANN example: car insurance frequencies

	# param.	in-sample loss (in 10^{-2})	out-of-sample loss (in 10^{-2})
homogeneous ($\mu \equiv \text{const.}$)	1	32.935	33.861
Model GLM (Poisson)	48	31.257	32.149
CANN (2-dim. embeddings)	792 (+48)	30.476	31.566

Note for low frequency examples of, say, 5%: we have in the true model $\mathcal{L}_{\mathcal{D}} \approx 30.3 \cdot 10^{-2}$.





Variants of CANN

- Freeze $\widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}$ (use as offset) and only train network parameter $\theta = (\theta_{1:d}, \theta_{d+1})$

$$\mu^{\mathrm{CANN}}_{(\boldsymbol{\beta},\theta)}(\boldsymbol{x}) = \exp\left\{\langle\widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}, \boldsymbol{x}\rangle + \left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x})\right\rangle\right\}.$$

- Introduce trainable credibility weight α for the offset

$$\mu^{\mathrm{CANN}}_{(\boldsymbol{\beta},\theta)}(\boldsymbol{x}) = \exp\left\{\alpha\langle\widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}, \boldsymbol{x}\rangle + (1-\alpha)\left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x})\right\rangle\right\}.$$

- Find missing interactions in (x_l, x_k) in addition to the offset

$$\mu^{\mathrm{CANN}}_{(\boldsymbol{\beta},\theta)}(\boldsymbol{x}) = \exp\left\{\langle\widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}, \boldsymbol{x}\rangle + \left\langle \theta_{d+1}, \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(x_l, x_k)\right\rangle\right\}.$$

Regularization step for the balance property

- Neural network calibrations do not have the balance property, yet.

- Apply an additional GLM step on the learned representation

$$\boldsymbol{x} \;\mapsto\; \boldsymbol{z} = \boldsymbol{z}(\boldsymbol{x}) = \left(\boldsymbol{z}^{(d)} \circ \cdots \circ \boldsymbol{z}^{(1)}\right)(\boldsymbol{x}),$$

  keeping the offset $\langle \widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}, \boldsymbol{x}\rangle$ fixed, i.e. calculate MLE $\widehat{\theta}_{d+1}^{\mathrm{MLE}}$ of regression function

$$\boldsymbol{z} = \boldsymbol{z}(\boldsymbol{x}) \;\mapsto\; \exp\left\{\langle \widehat{\boldsymbol{\beta}}^{\mathrm{MLE}}, \boldsymbol{x}\rangle + \langle \theta_{d+1}, \boldsymbol{z}\rangle\right\}.$$

- This regularization step is important, in particular, in classification problems having the class imbalance problem!

Summary and outlook

- CANN allows us to identify missing structure in GLMs (more) explicitly.
- An additional GLM step allows us to satisfy the balance property.
- CANN allows us to learn across different portfolios.

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