Risk-based versus target-based portfolio strategies in the cryptocurrency market

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Motivation

Correlations between traditional assets and cryptos

Figure 1: Correlations between cryptos and conventional financial assets, daily returns

Portfolio allocation strategies with CC
## Financial relevance

<table>
<thead>
<tr>
<th>Year</th>
<th>Price USD</th>
<th>Simple return (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>4.99</td>
<td>+1,565.01</td>
</tr>
<tr>
<td>2012</td>
<td>13.59</td>
<td>+172.07</td>
</tr>
<tr>
<td>2013</td>
<td>739.10</td>
<td>+5,338.56</td>
</tr>
<tr>
<td>2014</td>
<td>317.40</td>
<td>−57.06</td>
</tr>
<tr>
<td>2015</td>
<td>428.00</td>
<td>+34.85</td>
</tr>
<tr>
<td>2016</td>
<td>952.15</td>
<td>+122.47</td>
</tr>
<tr>
<td>2017</td>
<td>14,165.57</td>
<td>+1,387.75</td>
</tr>
<tr>
<td>(2018)</td>
<td>6,973.69</td>
<td>−50.77</td>
</tr>
</tbody>
</table>

Table 1: Recently high realised returns: BTC
Challenge of crypto investment: high risk

Figure 2: Crypto currencies have higher volatilities than stocks, highlighting the importance of risk management when investing to them.

Portfolio allocation strategies with CC
Challenge of crypto investment: low trading volume

Figure 3: Crypto currencies have much lower trading volume compared to traditional assets
Benefit of crypto investment

Figure 4: Markowitz portfolios with cryptos and without cryptos

Portfolio allocation strategies with CC
Challenge of crypto investment

Figure 5: Markowitz portfolio date with cryptos and without cryptos

Portfolio allocation strategies with CC
Is Markowitz rule appropriate?

Figure 6: Density of Top-10 cryptos against normal distribution (time span is 2015-01-01 to 2017-12-31.)
Cryptos from an investment viewpoint

- Elendner et al. (2016) & Yermack (2014): Cryptos show low correlation with traditional assets
- Chen et al. (2016): Analyzing dynamics of CRIX
- Trimborn, Li and Härdle (2018): Liquidity constrained risk-return portfolios in crypto markets
- Lee, Li and Wang (2018): Risk and return characteristics using portfolios with CRIX constituents
Motivation

Objectives

- Out-of-sample performance analysis – is there the best individual asset allocation model?
- Diversification of models – do the combinations of models outperform individual ones?
- Do portfolio and risk concentration depend on the investor objective function?
- Do liquidity constraints affect the portfolio performance?
- Diversification effects of inclusion of CCs in various concepts of diversification
Outline

1. Motivation ✓
2. Methodology
3. Data
4. Empirical results
5. Conclusion

Portfolio allocation strategies with CC
### Investment strategies

<table>
<thead>
<tr>
<th>Model</th>
<th>Reference</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally weighted</td>
<td>DeMiguel et al. (2009)</td>
<td>EW</td>
</tr>
<tr>
<td><strong>Risk-return-oriented strategies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean – Var – max Sharpe</td>
<td>Jagannathan and Ma (2003)</td>
<td>MV – S</td>
</tr>
<tr>
<td><strong>Return-oriented strategies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk – Return – max return</td>
<td>Markowitz (1952)</td>
<td>RR – max ret</td>
</tr>
<tr>
<td><strong>Risk-oriented strategies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean – Var – min var</td>
<td>Merton (1980)</td>
<td>MinVar</td>
</tr>
<tr>
<td>Equal Risk Contribution</td>
<td>Roncalli et al. (2010)</td>
<td>ERC</td>
</tr>
<tr>
<td>Mean – CVaR – min risk</td>
<td>Rockafellar and Uryasev (2000)</td>
<td>MinCVaR</td>
</tr>
<tr>
<td>Maximum Diversification</td>
<td>Rudin and Morgan (2006)</td>
<td>MD</td>
</tr>
<tr>
<td><strong>Combination of models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naïve combination</td>
<td>Schanbacher (2015)</td>
<td>COMB NAÏVE</td>
</tr>
<tr>
<td>Combination bootstrap</td>
<td>Schanbacher (2014)</td>
<td>COMB</td>
</tr>
</tbody>
</table>

Table 2: List of asset allocation models
Maximum diversification portfolio with Portfolio diversification index (PDI)

\[
\max_{w \in \mathbb{R}^N} \quad \text{PDI}_P(w) \\
\text{s.t.} \quad w^\top 1_N = 1, \\
\quad w_i \geq 0
\]

\[
\text{PDI}_P(w) = 2 \sum_{i=1}^{N} iW_i - 1,
\]

where \( W_i = \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i} \) are the relative strengths of the \( i \)-th principal portfolio.

Portfolio allocation strategies with CC
Investment universe

- 16 traditional assets
  - Equity indices: S&P100, FTSE100, SSE, NIKKEI225, SX5E
  - 10 Years government bonds: EU, UK, JP, CN, USA
  - Real estate & commodities: GOLD, MSCI ACWI COMMOD PRODUCERS, FTSE EPRA (NAREIT DEV REITS)
  - FIAT: EUR, GBP, CNY, YEN
- 55 crypto-currencies (97%/ 61% of Entire Market Cap)
- Sources: thecrix.de, Bloomberg
- Time span 2015-01-01 to 2017-12-31 (781 trading days/24 moving windows)

Portfolio allocation strategies with CC
Investment Universe

Figure 7: 55 crypto-currencies’ cumulative return compared with the initial investment – thick red line (returns are 95% winsorized)

Portfolio allocation strategies with CC
Efficient frontiers: significant shift with CC

Figure 8: Efficient frontiers build on daily basis: CCPEfficient_surface
Performance of portfolio strategies

Figure 9: Cumulative wealth: S&P100, EW, MV-S–TrA, EW–TrA and corresponding Allocation strategy

Portfolio allocation strategies with CC
Empirical results

Performance of portfolio strategies – LIBRO

Figure 10: Cumulative wealth ($M = 10^6$ US$): S&P100, EW, MV-S–TrA, EW–TrA and corresponding Allocation strategy CCPPerformance. Portfolio allocation strategies with CC
Capital allocation

Figure 11: Dynamics in the capital composition w/o liquidity constraints

Portfolio allocation strategies with CC

CCPWeights
Empirical results

**Capital allocation - LIBRO**

Figure 12: Dynamics in the capital composition ($M = 10^7$ US$)

Portfolio allocation strategies with CC

CCPWeights
Empirical results

Portfolio risk allocation

Figure 13: Dynamics of risk contributions for portfolio strategies with CC

Portfolio allocation strategies with CC
Empirical results

Portfolio risk allocation - LIBRO

Figure 14: Dynamics of risk contributions for portfolio strategies \( (M = 10^7 \text{ US$}) \)

Portfolio allocation strategies with CC

CCPRisk_contribution
Risk contribution and capital composition: results

- Very significant disparities in risk contributions and capital composition between different rules
- (Almost) no cryptos in global non-constraint minimum risk portfolio
- LIBRO approach affects risk and capital composition of portfolios
## Empirical results

### Portfolios' performance

<table>
<thead>
<tr>
<th>Allocation Strategy</th>
<th>CW</th>
<th>SR</th>
<th>ASR</th>
<th>CEQ</th>
<th>TURNOVER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No const</td>
<td>10 mln</td>
<td>No const</td>
<td>10 mln</td>
<td>No const</td>
</tr>
<tr>
<td>S&amp;P100</td>
<td>1.261</td>
<td>0.080</td>
<td>0.079</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>EW TrA</td>
<td>1.069</td>
<td>0.048</td>
<td>0.047</td>
<td>0.004</td>
<td>4.824</td>
</tr>
<tr>
<td>MV-S TrA</td>
<td>1.052</td>
<td>0.068</td>
<td>0.068</td>
<td>0.000</td>
<td>1.359</td>
</tr>
<tr>
<td>EW</td>
<td>3.644</td>
<td>0.132</td>
<td>0.132</td>
<td>0.004</td>
<td>1.102</td>
</tr>
<tr>
<td>MinVar</td>
<td>1.001</td>
<td>0.065</td>
<td>0.065</td>
<td>0.001</td>
<td>3.924</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>1.024</td>
<td>0.048</td>
<td>0.048</td>
<td>0.000</td>
<td>5.987</td>
</tr>
<tr>
<td>ERC</td>
<td>1.558</td>
<td>0.158</td>
<td>0.157</td>
<td>0.001</td>
<td>1.167</td>
</tr>
<tr>
<td>MD</td>
<td>5.147</td>
<td>0.158</td>
<td>0.158</td>
<td>0.007</td>
<td>4.408</td>
</tr>
<tr>
<td>RR-Max ret</td>
<td>4.703</td>
<td>0.003</td>
<td>0.003</td>
<td>0.000</td>
<td>0.229</td>
</tr>
<tr>
<td>MV-S</td>
<td>1.214</td>
<td>0.119</td>
<td>0.125</td>
<td>0.000</td>
<td>3.211</td>
</tr>
<tr>
<td>COMB NAÏVE</td>
<td>2.613</td>
<td>0.126</td>
<td>0.127</td>
<td>0.003</td>
<td>2.281</td>
</tr>
<tr>
<td>COMB</td>
<td>3.542</td>
<td>0.126</td>
<td>0.125</td>
<td>0.004</td>
<td>0.881</td>
</tr>
</tbody>
</table>

Table 3: Performance measures for monthly rebalancing frequency ($l = 21$)

Portfolio allocation strategies with CC
Difference of SR and CEQ

Table 4: p-value of the difference between the SR (lower triangle) and CEQ (upper triangle) of all strategies with each other with significance codes 0.01, 0.05 and 0.1 (without liquidity constraints)

Portfolio allocation strategies with CC
## Portfolios strategies: diversification effects

<table>
<thead>
<tr>
<th>Allocation Strategy</th>
<th>$DR^2$</th>
<th>Effective $N$</th>
<th>PDI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No const</td>
<td>10 mln</td>
<td>No const</td>
</tr>
<tr>
<td>MV - S TrA</td>
<td>5.70</td>
<td>5.70</td>
<td>3.37</td>
</tr>
<tr>
<td>RR - Max ret</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>MinVar</td>
<td>13.65</td>
<td>13.51</td>
<td>3.48</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>15.22</td>
<td>14.90</td>
<td>4.07</td>
</tr>
<tr>
<td>ERC</td>
<td>11.42</td>
<td>12.36</td>
<td>17.63</td>
</tr>
<tr>
<td>MD</td>
<td>2.99</td>
<td>2.41</td>
<td>4.08</td>
</tr>
<tr>
<td>MV -S</td>
<td>9.05</td>
<td>9.36</td>
<td>3.70</td>
</tr>
<tr>
<td>COMB NAÏVE</td>
<td>3.95</td>
<td>4.58</td>
<td>12.55</td>
</tr>
<tr>
<td>COMB</td>
<td>4.09</td>
<td>4.26</td>
<td>8.58</td>
</tr>
</tbody>
</table>

*All diversification measures are calculated based on in-sample data and averaged over the period 20150101-20171130.

Table 5: Measures of diversification for monthly rebalancing
Portfolio allocation strategies with CC
Conclusion I

- Out-of-sample performance:
  - MD and Max Return strategies show the most promising results
  - CC as portfolio components yield little variance reduction: application of CC in target return portfolio strategies
  - The inclusion of CC is strongly related to investment objectives (utility function)

- Bootstrap combination of models outperforms individual ones in many aspects
Conclusion II

- Capital and risk portfolio compositions are not robust
- LIquidity Bounded Risk-return Optimization (LIBRO) approach for CC portfolios:
  - improves risk-adjusted performance
  - strengthens diversification effects
- CC enhance diversification benefits in comparison with only conventional assets’ portfolio
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Mean-Variance Asset Allocation

Log returns $X_t \in \mathbb{R}^p$:

$$\min_{w \in \mathbb{R}^p} \quad \sigma_P^2(w) \stackrel{\text{def}}{=} w^\top \Sigma w$$

s.t. $\mu_P(w) = r_T,$

$$w^\top 1_N = 1, \quad w_i \geq 0$$

where $\Sigma \stackrel{\text{def}}{=} E_{t-1}\{(X - \mu)(X - \mu)^\top\}$ is the sample covariance matrix of returns, $\mu_P(w) \stackrel{\text{def}}{=} w^\top \mu$, $\mu \stackrel{\text{def}}{=} E_{t-1}(X)$ is the portfolio mean and $r_T$ – "target" return

Portfolio allocation strategies with CC
Risk Parity (Equal risk contribution)

\[ \sigma_P(w) = \sqrt{w^\top \Sigma w} \] is the Euler decomposition of volatility, then:

\[ \sigma_P(w) \overset{\text{def}}{=} \sum_{i=1}^{N} \sigma_i(w) = \sum_{i=1}^{N} w_i \frac{\partial \sigma_P(w)}{\partial w_i} \] (4)

where \( \frac{\partial \sigma_P(w)}{\partial w_i} \) is the marginal risk contribution and \( \sigma_i(w) = w_i \frac{\partial \sigma_P(w)}{\partial w_i} \) is the risk contribution of \( i \)-th asset. In ERC portfolio:

\[ \sigma_i(w) = \sigma_j(w) = \frac{1}{N} \] (5)
Conditional VaR optimization

Given $\alpha < 0.05$ risk level, the CVaR optimized portfolio weights $w$ are calculated as:

$$\min_{w \in \mathbb{R}^N} \text{CVaR}_\alpha(w), \quad \text{s.t. } \mu_P(w) = r_T, \ w^\top 1_p = 1, \ w_i \geq 0, \quad (6)$$

$$\text{CVaR}_\alpha(w) = -\frac{1}{1 - \alpha} \int_{w^\top X \leq -\text{VaR}_\alpha(w)} w^\top X f(w^\top X|w) \, dw^\top X, \quad (7)$$

where $\frac{\partial}{\partial w^\top X} F(w^\top X|w) = f(w^\top X|w)$ is pdf of the portfolio returns portfolio weights $w$. $\text{VaR}_\alpha(w)$ is $\alpha$-quantile of the cdf.
Averaging of portfolio models

Consider $m$ asset allocation models with weights $W_t = (w_t^1 \ldots w_t^m)$, then individual shares:

$$\pi = (\pi^1 \ldots \pi^m),$$

s.t. $\pi^\top 1_m = 1$ (8)

the combined portfolio weight

$$w^{comb} = \sum_{i=1}^{m} \pi^i w^i$$ (9)

Naïve combination: $\pi^i = \frac{1}{m}$ for all $i = 1 \ldots m$
Averaging of portfolio models: bootstrap approach

The probability that model \( i \) outperforms all other models:

\[
\hat{\pi}^i = \frac{1}{B} \sum_{b=1}^{B} s_{i,b}
\]  

\[s_{i,b} = \begin{cases} 
1, & \text{if } l_{i,b} > l_{j,b} \text{ for } i \neq j \\
0, & \text{otherwise}
\end{cases}
\]

\( B \) - number of independent bootstrap samples

\( l \) - loss function optimizing CEQ: \( l(w) = w^T \hat{\mu} - \frac{\gamma}{2} w^T \hat{\Sigma} w \)
Liquidity Bounded Risk-return Optimization (LIBRO) I

Daily Trading Volume (TV):

\[ TV = \sum_{i=1}^{n} p_i \cdot q_i \]  

- \( n \) - the number of trades
- \( p_i \) - the price of assets of trade \( i \)
- \( q_i \) - the number of assets of trade \( i \)
the market value held in asset $i$

$$M w_i \leq TV_i \cdot f_i,$$  \hspace{1cm} (13)

$f_i$ - controls the speed of clearing the position on asset $i$

$M$ - investment amount

$$w_i \leq \frac{TV_i \cdot f_i}{M} = \hat{a}_i$$  \hspace{1cm} (14)
\[ w_i \leq \frac{TV_i \cdot f_i}{M} = \hat{a}_i \quad (15) \]

\[ \hat{a}_i = \frac{\hat{\text{Liq}}_i}{M} \cdot c \quad (16) \]

- \( \hat{\text{Liq}}_i = TV_i \cdot f_i \) is a liquidity, \( \hat{\text{Liq}}_i \in \mathbb{R}_0^+ \setminus \{\infty\} \) for asset \( i \)

- \( c \) is the factor controlling the amount of permitted short-selling
LINRO IV

MV LIBRO optimization problem is then:

$$\min_{w \in \mathbb{R}^p} \quad \sigma_P^2(w) \overset{\text{def}}{=} w^\top \Sigma w$$

s.t. \quad \mu_P(w) = r_T, \quad w^\top 1_p = 1, \quad 0 \leq w_i \leq \hat{a}_i \quad (17)$$
Evaluation of Portfolios’ Performance

- Certainty-EQuivalent (CEQ) return

\[ \hat{CEQ}_{i,\gamma} = \hat{\mu}_i - \frac{\gamma}{2} \hat{\sigma}_i^2 \]  

(18)

where \( \gamma \) reflects the investor’s risk aversion

- Turnover

\[ Turnover_i = \frac{1}{T-L} \sum_{t=1}^{T-L} \sum_{j=1}^{N} |\hat{w}_{j,t+1} - \hat{w}_{j,t+1}| \]  

(19)

where \( w_{j,t+1} \) is the portfolio weight before rebalancing at \( t + 1 \),

\( L \) - length of moving window
Evaluation of Portfolios’ Performance

- **Sharpe Ratio (SR)**
  \[
  SR_i = \frac{\hat{\mu}_i}{\hat{\sigma}_i^2}
  \]  
  (20)

- **Adjusted Sharpe Ratio (ASR)**
  \[
  ASR_i = SR_i \left[ 1 + \left( \frac{S}{6} \right) SR_i - \left( \frac{K}{24} \right) SR_i^2 \right]
  \]  
  (21)

  where \( S \) - skewness and \( K \) - excess kurtosis.
**P-values**

Ledoit ans Wolf (2008)

Let $X_i$ and $X_j$ - returns produced by strategies $i$ and $j$ and

$$\nu = (\mu_i, \mu_j, E(X_i^2), E(X_j^2))^\top$$

Difference of CEQ and SR

$$f_{CEQ}(\nu) = \mu_i - \frac{\gamma}{2} (E(X_i^2) - \mu_i^2) - \mu_j + \frac{\gamma}{2} (E(X_j^2) - \mu_j^2)$$

$$f_{SR}(\nu) = \frac{\mu_i}{\sqrt{E(X_i^2) - \mu_i^2}} - \frac{\mu_j}{\sqrt{E(X_j^2) - \mu_j^2}} \quad (22)$$
P-values ctd.

- Delta method: if $\sqrt{T-L(\hat{\nu} - \nu)} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Psi)$, then

$$\sqrt{T-L(\hat{f} - f)} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \nabla^\top f(\nu)\Psi\nabla f(\nu)),$$  \hspace{1cm} (23)

where $\nabla f$ is a derivative of $f$

- Standard Error for $\hat{f}$:

$$SE(\hat{f}) = \sqrt{\frac{\nabla^\top f(\nu)\Psi\nabla f(\nu)}{T-L}}$$  \hspace{1cm} (24)

- Solutions for consistent estimator for $\hat{\Psi}$: HAC and Bootstrap inference

Portfolio allocation strategies with CC
P-values ctd.

- HAC inference

\[
\Psi_{T-L} = \frac{T-L}{T-L-4} \sum_{j=-T+L+1}^{T-L-1} \frac{n}{S_{T-L}} \hat{\Gamma}_{T-L}(n)
\]  

(25)

\[
\hat{\Gamma}_{T-L}(j) = \begin{cases} 
\frac{1}{T-L} \sum_{t=n+1}^{T-L} \hat{y}_t \hat{y}_{t-n}^\top & \text{for } j \geq 0 \\
\frac{1}{T-L} \sum_{t=-n+1}^{T-L} y_{t+n} \hat{y}_t^\top & \text{for } j \geq 0 
\end{cases}
\]

(26)

- \(\hat{y}^\top t = (x_{ti} - \hat{\mu}_i, x_{tj} - \hat{\mu}_j, x_{ti}^2 - E(X_i^2), x_{tj}^2 - E(X_j^2))\)

- \(k(\cdot)\) is a kernel, \(S_{T-L}\) is a bandwidth

Back to "P-values"
A two-sided $p$-value for $H_0: f = 0$

$$\hat{p} = 2\Phi \left| \hat{f} \right| / \text{SE}(\hat{f})$$

(27)
Measures of diversification

- **Effective N**

\[
N_{Eff}(w) = \frac{1}{\sum_{i=1}^{N} w_i^2}
\]  

(28)

- **Diversification Ratio (DR)**

\[
DR(w) = \frac{w^\top \sigma}{\sqrt{w^\top \Sigma w}} = \frac{w^\top \sigma}{\sigma_P(w)}
\]

(29)

Back to "Results"
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