Correlation scenarios and correlation stress testing

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Overview

- **Correlation** lies at the heart of many financial applications: portfolio risk-management, diversification, hedging.

- Principal idea: link *economically meaningful scenarios* to *correlation scenarios*

Overview

Current **working paper**: Packham, N. and Woebbecking, F.: *Correlation scenarios and correlation stress testing*. [link]

**Objectives:**

- **Correlation factor model** for any kind of financial asset portfolio
- **Bayesian factor selection** to incorporate a priori knowledge
- **Stress testing**: portfolio effect of adverse correlation scenarios
- **Reverse stress testing**: identify extreme yet plausible scenarios

Motivation
The “London Whale”

▶ “London Whale”: 2012 Loss at JPMorgan Chase & Co. of approx. 6.2 bn USD on a credit derivatives portfolio

▶ Authorised trading position, hence risk management problem

▶ Synthetic credit portfolio (SCP): 120 long and short positions, CDX and iTraxx index + tranche products, investment grade and high-yield

▶ “Smart short” strategy: credit protection on high yield is financed by selling protection on investment grade indices.

▶ Timeline:
  – End of 2011: decision to reduce SCP’s risk-weighted assets (RWA’s).
  – Avoid liquidation costs by increasing positions with opposite market sensitivity (hedges).
  – 23 March 2012: Senior executives ordered to stop trading on SCP; net notional of 157 bn USD (up 260% from September 2011).

▶ Risk management of SCP focussed on value-at-risk (VaR) and CSW-10 (credit spread widening of 10 basis points).

▶ Publicly available information: JPMorgan, 2013; United-States-Senate, 2013a,b
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Motivation

Methodology
  Correlation parameterisation
  Factor selection
  Stress testing

Application (equity portfolio)

Conclusion
Interest-rate modelling: Correlation parameterisation

- Parametric correlation models widespread in interest-rate modelling / LIBOR market model, e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2000); Packham (2005)

- Simplest case: Correlation $c_{ij}$ between two forward LIBOR’s is given by

$$c_{ij} = e^{-\beta|i-j|},$$

where $\beta > 0$ is a parameter, and $i, j$ represent maturities.

- Captures stylised fact that correlations decay with increasing maturity difference
Link correlations to risk factors

- Idea: Carry over “distance” measure to other risk factors, such as geographic regions, industries, investment grade vs. high-yield, ...

- Association of asset $i \in \{1, \ldots, p\}$ with factor $k \in \{1, \ldots, d\}$:

$$1_{\{k,i\}}$$

[Assume this as given for the time being.]

- Correlation parameterisation:

$$c_{ij} = \tanh\left(\sum_{k=1}^{d} \lambda_k |1_{\{k,i\}} - 1_{\{k,j\}}| + \sum_{k=1}^{d} \nu_k 1_{\{k,i\}} 1_{\{k,j\}}\right),$$

with coefficients $\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d \in \mathbb{R}$. 

Methodology
Link correlations to risk factors

- \( \tanh : \mathbb{R} \rightarrow [-1, 1] \) allows for negative correlations.
- \( \tanh \) used in inferential statistics on sample correlation coefficients (\( \leadsto \) Fisher transformation).
- The following summation formula is helpful for a rough interpretation of the coefficients:

\[
\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}
\]

![Graph of \( \tanh(x) \)]
Correlation parameterisation

- Given a sample correlation matrix at one time point, the coefficients $\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d$ can be determined e.g. by ordinary least squares on $\text{arctanh}(c_{ij})$, the inverse of $\tanh$.

- Simple correlation scenarios such as “the correlation between assets exposed to factor $k$ and assets not exposed to factor $k$ increases” is then implemented by increasing $\lambda_k$ (e.g. Europe vs US).

- Likewise, a scenario such as “the correlation of firms exposed to factor $k$ increases” is implemented by increasing $\nu_k$ (e.g. within Europe).

- With parameters calibrated on a regular basis, the parameter history can be used to obtain realistic scenarios (reverse stress test).
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Principal ideas

- Risk factors in “London Whale” were tailored to specific portfolio.
- In practice, factor models use **industries** and **countries** as factors to model asset correlations.
- **Problem:** How to assign factors to assets?
  - Number of factors should be **small**, but include all **important** factors.
  - **Prior information:** country of firm’s headquarter, primary industry
  - **Bayesian variable selection** to determine small number of factors driving asset return
Different methods, e.g.

- **Bayesian model selection** compares posterior probabilities of different models.

- **Spike and slab priors** include an indicator variable for each coefficient and determines the indicator variable’s posterior probability of taking value one.

In our setting, **Bayesian model selection** worked best.
Bayesian model selection

- Denote candidate models by $M_i$, $i = 1, \ldots, m$.
- In a linear regression setting, each model $M_i$ includes a specific subset of independent variables (potential risk factors) and excludes the other variables.
- **Posterior model probability:**

\[ p(M_i|y) \propto p(y|M_i)p(M_i), \]

where
- $y$ is the time series of a firm's asset returns,
- $p(M_i)$ is the prior model probability,
- $p(y|M_i)$ is called the marginal likelihood.

(see e.g. Appendix B.5.4 of (Fahrmeir et al., 2013))
Bayesian model comparison

- **Posterior inclusion probabilities (PIP):**

\[
P(1_{\{\beta_k \neq 0\}} = 1|y) = \sum_{\beta_k \in M_i} P(M_i|y).
\]

- If number of parameters $p$ is large, then full calculation of $2^p$ posterior model probabilities is infeasible.

⇒ Use **Markov Chain Monte Carlo (MCMC)** simulation.

- Factors with PIP greater 0.5 are selected
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Stress-testing correlations

- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by **value-at-risk (VaR)** in variance-covariance approach:

\[
\text{VaR}_\alpha = -V_0 \cdot N_{1-\alpha} \cdot (w^T \Sigma w)^{1/2},
\]

with
- current position value \(V_0\),
- \(N_{1-\alpha}\): \((1 - \alpha)\)-quantile of the standard normal distribution,
- vector of portfolio weights \(w\) and
- covariance matrix \(\Sigma\).

- For **correlation stress test**, only need to consider portfolio variance

\[w^T \Sigma w.\]
Revere stress testing

- What is the **worst scenario** amongst all scenarios that occur within some pre-given **range**?

- Restrict **risk-factor distribution** \((\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)\)

- **Univariate setting**: quantile

- **Multivariate setting**:
  - Mahalanobis distance (Mahalanobis, 1936),
  - highest density regions (HDR) (Hyndman, 1996a),
  - concepts based on norms, e.g. (Serfling, 2002).

- Maha is closely tied to the normal or to elliptical distributions.

- HDR allows for more flexibility (e.g. skewness and tail heaviness).
Highest density region (HDR)

- Let \( f(x) \) be the density function of a random vector \( X \)

- The \( 100(1 - q)\% \) HDR is the subset of \( R(f_q) \) of the sample space of \( X \) such that

\[
R(f_q) = \{ x : f(x) \geq f_q \}
\]

where \( f_q \) is the largest constant such that \( P(X \in R(f_q)) \geq 1 - q \).

- Worst-case scenario within given HDR:

\[
\beta^* = \arg\max_{\beta \in R(f_q)} \text{VaR}_\alpha(\beta).
\]

(Hyndman, 1996b)
Overview

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Methodology

Application (equity portfolio)
  Factor selection and fit
  Stress test

Conclusion
Factor selection

- Factors: MSCI stock indices representing 6 geographic regions and 11 industries
- Individual stocks: 505 S&P constituents, 30 DAX constituents
- Daily data from 1999-Jan 2021 (Source: Bloomberg, MSCI, Reuters)
- Factor assignment re-calibrated every quarter, based on 3-years of daily data (88 quarters)
- Prior: hard-code primary geographic region and industry,
- 6 factors on expectation
Factor selection

- Number of quarters that each factor is included for SAP
- German IT company

Application (equity portfolio)
Factor selection

- Number of quarters that each factor is included for Amazon:
- US based online retailer with strong presence in Europe
- World’s largest provider of computing services (AWS)

![Diagram showing factor selection]

Application (equity portfolio)
Correlations at beginning of Covid-19 pandemic


Application (equity portfolio)
Factor coefficients

Selected parameter time series (grey vertical lines indicate variable selection intervals)

MM-Americas
intra_Financials
intra_MM-Americas

▶ Fitted parameters for risk factors with high loads.

Application (equity portfolio)
Factor coefficients

Fitted “intra” parameters for selected risk factors (\(\nu_k\))
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Fit time series of risk factor parameters \((\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)\) to Normal-Inverse Gaussian (NIG) distribution

NIG: generalisation of normal dist. that allows for skewness and higher variation in tails

Calibration via using expectation-maximization (EM) algorithm, (McNeil et al., 2005, Chapter 3) and Dempster et al. (1977)
Value-at-risk impact

- **Blue**: $\text{VaR}_{99\%}^{1\text{ day}}$ on equally-weighted portfolio of DAX and S&P 500
- **Orange**: Stressed $\text{VaR}_{99\%}^{1\text{ day}}$ on reverse stress scenario of 5 April 2021.

Application (equity portfolio)
Reverse stress testing (Covid-19 pandemic)

HDR Boxplots of Correlation Parameters

- Worst-case scenario within 95% HDR (18 Feb 2020)
- Triangles: worst-case scenarios (MC sim., Hist. sim.)
- Stars: Scenarios on 18 Feb (green) and 18 March (blue)

Application (equity portfolio)
Conclusion

- We develop a correlation stress testing framework, linking risk factors with correlations.
- Risk factors (e.g. industries, countries) are linked firms via Bayesian variable selection methods.
- Reverse stress tests are conducted by assigning the factor loadings a distribution and determining the worst-case scenario within a HDR.
References I


References II


Thank you!

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